

9.1 Introduction to Polynomials and Polynomial Functions

You already have an extensive knowledge of algebraic expressions, but we will look in depth at important algebraic expressions known as **polynomials**.

A **term** can be a number (like -3), a variable (like x), a product or quotient of numbers and/or variables (like $-3x$, $2xy^2z$, or $4/t$).

A **monomial** is a product of constants and/or variables. ***Note monomials can not include division by a variable.*** The sum of monomials is called a **polynomial**.

Examples of Polynomials: $3a + 3$, $\frac{1}{2}x^2$, $5x^3y^2$

Not examples of Polynomials: $\frac{x+3}{x-2}$, $3x^2 - 2y^2 + \frac{1}{x}$, $\frac{1}{x^2-2}$

When a polynomial is written as a sum of monomials, each monomial is called a **term of the polynomial**.

We classify polynomials by how many terms it has.

MONOMIALS	BINOMIALS	TRINOMIALS	NO SPECIAL NAME
$3x^2$	$2x + 3$	$3t^2 + 4t + 7$	$4x + 3y + 2z + 1$
9	$3a^2 - b$	$6x^7 - 5z^3 + 2y$	$5p^5 - 4p^4 + 3p^3 + 2p^2 + 1$
$-4a^{12}b^4$	$-6x^7 + 5y$	$5a - 6b^2 - \frac{1}{2}$	$8x^4 + 7x^5 - 2x^6 + 5x$

Degree and Coefficients

The **degree** of a term of a polynomial is the sum of the exponents of the variables that term. For example:

- 1.) $9x^4$ has a degree of 4.
- 2.) $2x$ has a degree of 1.
- 3.) $3x^3yz^2$ has a degree of 6.
- 4.) 8 has a degree of 0.

The **coefficient** is the part of a term that is a constant factor.

Example: In the term $-7x^2y$, -7 is the coefficient.

The **leading term** of a polynomial is the term with the highest degree. Its coefficient is called the **leading coefficient** and its degree is referred to as the **degree of the polynomial**.

Consider the polynomial: $7x^2 - 9x^3 + 2x^5 + x - 2$

The leading term is $2x^5$, the leading coefficient is 2, and the degree of the poly is 5.

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We generally arrange the polynomial in one variable in **descending** degree order. Consider the previous polynomial $7x^2 - 9x^3 + 2x^5 + x - 2$. Write this polynomial in descending order.

$$7x^2 - 9x^3 + 2x^5 + x - 2 = 2x^5 - 9x^3 + 7x^2 + x - 2$$

Polynomials which have several variables can be arranged with respect to the powers of one of the variable

Polynomials can be written in function form and are classified by the degree of the polynomial.

TYPE OF FUNCTION	DEGREE	EXAMPLE
Linear	1	$f(x) = 2x - 5$
Quadratic	2	$g(x) = 3x^2$
Cubic	3	$p(x) = 4x^3 - 2x^2 - 3$
Quartic	4	$h(x) = 9x^4 - 1$