

7.6 Classic 2 variable Problems.

Age Puzzles: Example \rightarrow A father is 32 years older than his son. In 4 years the father will be 5 times older than his son. How old is each now?

Step 1: Define variables
Let f = father's age now
 s = son's age now

Step 2: Express statements as equations with variables

\rightarrow A father is 32 years older than his son

$$f = s + 32$$

\rightarrow In 4 years the father will be 5 times older than son

In other words... father in 4 years = 5 times son's age in 4 years
 $f + 4 = 5(s + 4)$
 $f + 4 = 5s + 20$
 $(f = 5s + 16)$

Step 3: There's our system

$$\begin{cases} f = s + 32 \\ f = 5s + 16 \end{cases}$$

$$\begin{cases} f = s + 32 \\ f = 4 + 32 \\ f = 36 \end{cases}$$

Substitution $\rightarrow 5s + 16 = s + 32$
 $4s = 16$
 $(s = 4)$

Speed with Wind and Current Puzzles

A plane leaves NYC and goes to Chicago, which is 750 miles away. Flying against the wind takes 2.5 hrs. Flying with the wind takes 2 hrs. Find the Speed of the plane and speed of wind.

Define variables

x = Speed of plane
 y = Speed of wind

rate \times time = distance

Flying to Chicago \rightarrow rate = $x - y$
Flying to NYC \rightarrow rate = $x + y$

$$2x = 675$$

$$x = 337.5 \text{ mi/hr}$$

$$y = 37.5 \text{ mi/hr}$$

Express equations

$$\begin{cases} (x - y) \cdot 2.5 = 750 \\ (x + y) \cdot 2 = 750 \end{cases} \rightarrow \begin{cases} x - y = 300 \\ x + y = 375 \end{cases}$$

elimination

Coin Puzzle Example: Coin bank has 250 dimes and quarters totalling \$39.25
How many dimes and quarters do you have.

Define variables
 d = dimes
 q = quarters

Equations with coins usually are:

1. Equation expressing number of coins
2. Equation expressing value.

$$\begin{cases} d + q = 250 \\ .10d + .25q = 39.25 \end{cases} \rightarrow \text{multiply both sides by 100 to get rid of decimals}$$

I decided to use elimination for this type of problem. You can easily have used substitution as well.

$$\begin{aligned} & \begin{cases} d + q = 250 \\ 10d + 25q = 3925 \end{cases} \\ & \begin{array}{r} -10d + -10q = -2500 \\ 10d + 25q = 3925 \\ \hline 15q = 1425 \\ q = 95 \end{array} \end{aligned}$$

$$\begin{aligned} & \begin{cases} d + q = 250 \\ d + 95 = 250 \end{cases} \\ & \quad d = 155 \end{aligned}$$

Chemical Solution Puzzle Example:

A 1.5% acid solution is mixed with a 4% acid solution.
How many ounces of each solution are needed to obtain 60 ounces of a 2.5% acid solution.

Define variables
 x = 1.5% Solution
 y = 4% Solution

$$\begin{cases} x + y = 60 \\ .015x + .04y = .025(60) \end{cases}$$

$$\begin{aligned} x + y &= 60 \rightarrow y = 60 - x \\ 15x + 40y &= 1500 \end{aligned}$$

$$\begin{aligned} 15x + 40(60 - x) &= 1500 \\ 15x + 2400 - 40x &= 1500 \\ -25x + 2400 &= 1500 \\ -25x &= -900 \\ x &= 36 \end{aligned}$$

$$\begin{aligned} x + y &= 60 \\ 36 + y &= 60 \\ y &= 24 \end{aligned}$$

Number Digit Puzzles

Important concept \rightarrow number in tens place is actually 10 times that number

$$35 = 3(10) + 5$$

$$53 = 5(10) + 3$$

$$451 = 4(100) + 5(10) + 1$$

Example: The sum of the digits of a two-digit number is 7. The original two-digit number is 3 less than 4 times the number with its digits reversed. Find the original two-digit number.

Define Variables
 t = tens digit
 u = units digit

$$\{ t + u = 7 \text{ or } t = 7 - u$$

Original number
 $10t + u$

Reverse $10u + t$

$$10t + u = 4(10u + t) - 3$$

$$10t + u = 40u + 4t - 3$$

$$\cancel{10t + u} \quad \underbrace{6t - 39u = -3}$$

Substitution?

$$\begin{cases} t = 7 - u \\ 6t - 39u = -3 \end{cases}$$

$$6(7 - u) - 39u = -3$$

$$42 - 6u - 39u = -3$$

$$42 - 45u = -3$$

$$-45u = -45$$

$$\underbrace{u = 1}$$

$$t = 7 - u$$

$$t = 7 - 1$$

$$\underbrace{t = 6}$$

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