6.5 Absolute-Value Equations and Inequalities

If we remember that the absolute value means the distance from zero on the number line then we must see that every absolute value equation could have two possible solutions.
Here's what I mean...

$|x| = 3$ We can think of this as "the distance from zero equals 3" Well there are two numbers with a distance of 3 aren't there? 3 and -3

Case 1 Case 2

$x = 3$ or $x = -3$

What about

$|x+6| = 4$, then Case 1 $\rightarrow$ $x+6=4$ or Case 2 $x+6=-4$
$x=-2$ or $x=-10$

one more...

$|3x-4| = 10$

Case 1 Case 2

$3x-4=10$ or $3x-4=-10$
$3x=14$
$x=14/3$ or $3x=-6$
$x=-2$

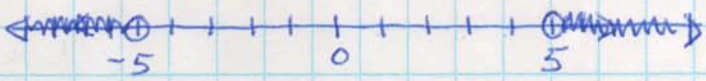
Inequalities are thought of in much the same way.

Ex: $|x+2| > 5$

ok think of the number line "the distance from zero is greater than 5" What does that look like?

Case 1 Case 2

$x+2 < -5$ or $x+2 > 5$
$x < -7$ or $x > 3$



That means what's in the $| \; |$ has to be less than -5 or greater than 5.

Let's try a few more examples

$|x+8|>1$ "distance greater than 1"

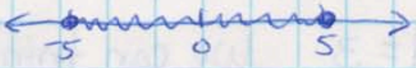
$x+8<-1$ or $x+1>1$ $\quad -1 \leftarrow$ and $1 \rightarrow$

$\boxed{x<-9}$ or $\boxed{x>0}$

$|2x+3| \leq 5$ "distance less than or equal to 5"

$2x+3 \geq -5$ and $2x+3 \leq 5$ $\quad A \geq -5$ and $A \leq 5$

$2x \geq -8$ and $2x \leq 2$



$x \geq -4$ and $x \leq 1$

So $\boxed{-4 \leq x \leq 1}$

CW. Pg 305 (5-17) all

H.W. Pg 305-306 (18-48 Evens)